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Electroweak interactions and high-energy limit [1]

An introduction to Equivalence Theorem

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Abstract

A pedagogical introduction to the equivalence theorem for longitudinal vector bosons in electroweak theories is presented and the problem of high-energy behaviour of scattering amplitudes in the Standard Model is briefly reviewed. To make the treatment self-contained, the basics of the Standard Model are summarized in an appendix.

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1 Introduction

Long since it has been known that in theories involving charged vector bosons with non-zero mass (i.e. typically in weak interaction models) the tree-level Feynman diagrams may diverge badly in the high-energy limit. The divergences are associated with physical states of spin -1 particles carrying longitudinal polarization, i.e. zero helicity. Such a divergent behaviour would in turn lead to rapid violation of the S-matrix unitarity in the tree approximation (similarly as in the old Fermi theory) unless there is a special mechanism suppressing the unwanted terms in physical scattering amplitudes (for an early reference concerning these matters see e.g. [1]). It is well known that the present-day standard model (SM) of electroweak interactions does provide such a mechanism: There are characteristic subtle cancellations among different Feynman graphs contributing to a considered S-matrix element, so that the tree-level physical scattering amplitudes are bounded in the high-energy limit (an example of such a non-trivial divergence cancellation was first observed by Weinberg [2]). The resulting partial-wave amplitudes then grow at most logarithmically, and the corresponding "unitarity bounds" are thus shifted to astronomically high values. What is even more important, such a good high-energy behaviour of scattering amplitudes at the tree level ("tree unitarity" [3]) is in fact a necessary condition for perturbative renormalizability. (Strictly speaking, there is no complete rigorous proof of this remarkable connection, but the usual arguments, based mostly on dispersion relations – see e.g. [3] – indicate that the statement is valid beyond any reasonable doubt; needless to say, there is also no known counter-example).

The above-mentioned delicate cancellations of diverging contributions coming from different Feynman diagrams may appear "miraculous" in the context of a straightforward calculation in the "physical" U-gauge. In fact, this spectacular phenomenon can be traced to the original (spontaneously broken) gauge symmetry, which is completely hidden in the U-gauge. The first general proof of the tree unitarity in spontaneously broken gauge theories was given by Bell [4]. A discussion of the inverse problem, which in a sense is even more interesting, followed immediately: In particular, Llewellyn Smith [5], and independently Cornwall, Tiktopoulos and Levin [3] and Joglekar [6]

[3]The technical term "tree unitarity" means that the n-particle S-matrix elements do not grow more rapidly than E^{4-n} in the $E \to \infty$ limit.

have shown (under some simple additional constraints) that a theory involving massive charged vector bosons, which is to satisfy the requirement of the tree unitarity, must be a non-abelian gauge theory with the Higgs mechanism realized by means of elementary scalars (for a pedagogical derivation of the SM along these lines, see e.g. [7]).

At present, a most powerful tool giving clear insight into the nature of the high-energy behaviour of the scattering amplitudes in SM (and in other models of this class) is the so-called Equivalence Theorem (ET) which relates a physical S-matrix element, involving external longitudinally polarized vector bosons, to its formal "unphysical counterpart" where the longitudinal vector bosons are replaced by the corresponding (unphysical) Higgs - Goldstone scalars (unphysical because they actually disappear from the physical spectrum as a result of the Higgs mechanism).

A statement of this kind has already been mentioned in [3] and several instructive examples were given by Vayonakis [8]. ET has subsequently been formulated by Lee, Quigg and Thacker [9] along with a sketch of the proof in the simplest case of one external longitudinal vector boson. The first attempt at a general proof is due to Chanowitz and Gaillard [10], and somewhat later it was put into a particularly nice and simple form by Gounaris, Kögerler and Neufeld [11]. These treatments were then improved by including properly the relevant renormalization factors arising beyond the tree level; such a program was started by Yao and Yuan [12], followed by Bagger and Schmidt [13], Kilgore [14], and completed by He, Kuang and Li in the series of papers [15]. Further aspects of ET are still investigated even in very recent literature, in particular in connection with the discussion of the mechanism of electroweak symmetry breaking both within and beyond SM (cf. e.g. [16], [17], [18], [19]). Some other papers dealing with the subject may be found under ref. [20]. One should also mention an earlier pedagogical treatment by Peskin [21] and the monograph [22] where this topic is also discussed.

The aim of the present lectures is to provide an introduction to the Equivalence Theorem, which undoubtedly represents one of the very remarkable aspects of modern gauge theories. We thus supplement partly the material of the book [7], devoted to the theme of divergence cancellations in scattering amplitudes at high energies within SM. In order to make these lecture notes rather self-contained, the conventional formulation of SM is summarized concisely in the Appendix.

2 R-gauges in the Standard Model

From the preliminary statement of the Equivalence Theorem mentioned briefly in the Introduction it is clear that for its formulation one has to use such a formal description of SM, in which the unphysical Higgs-Goldstone fields are preserved as auxiliary variables. This is achieved by using a class of the so-called renormalizable gauges (or simply R-gauges). The R-gauge technique, originally due to 't Hooft [23] was further developed by Fujikawa, Lee and Sanda [24] and nowadays it is widely used in practical Feynman diagram calculations in SM.

First, recall how the U-gauge is defined (see Appendix). Fixing the U-gauge means to eliminate completely the would-be (i.e. unphysical) Goldstone bosons which in a well-known sense are "natural partners" of the massive vector bosons emerging from the Higgs mechanism. (Setting the three Goldstone fields identically to zero is feasible since it is formally equivalent to a $SU(2)$ gauge transformation.) A nice feature of the U-gauge is that the corresponding interaction Lagrangian is relatively simple as it involves only physical fields (for a summary see e.g. Appendix K in [7]). However, there is a price to be paid for this convenience: The U-gauge propagator of a massive vector boson (W or Z) has the canonical form

$$D_{\mu\nu}(k) = \frac{-g_{\mu\nu} + m^{-2}k_\mu k_\nu}{k^2 - m^2 + i\varepsilon} \tag{1}$$

where m is a corresponding mass. Obviously, it does not decrease for $k \to \infty$ like the "normal" propagators do, and this in turn leads to a highly divergent behaviour of Feynman diagrams.

The R-gauges were invented in order to tame the severe divergences of Feynman diagrams in non-abelian gauge theories with Higgs mechanism. Fixing an R-gauge is completely different from the U-gauge case. To define it, one keeps the would-be Goldstone bosons in the game as auxiliary unphysical fields; one then adds to the original gauge invariant Lagrangian a non-invariant "gauge-fixing term" involving both vector fields and the unphysical scalars in the manner similar to the familiar "Fermi trick" in QED (which consists in adding a term proportional to $(\partial \cdot A)^2$ to the Maxwell Lagrangian).

Let us now specify the outlined procedure more precisely. The original complex Higgs doublet is conveniently parametrized as

$$\Phi = \begin{pmatrix} -iw^+ \\ \frac{1}{\sqrt{2}}(v + H + iz) \end{pmatrix} \tag{2}$$

where w^+ is complex and H and z are real (the constant v has the usual meaning). Note that by shifting the lower component of the doublet one gets mass terms for vector bosons (and for the H, of course), so one may define $W^{\pm}_{\mu}$, Z_{μ}, A_{μ} instead of $\overrightarrow{A_{\mu}}$, B_{μ} as usual (see (A.7), (A.9)). The gauge-fixing term is taken to be

$$\mathcal{L}_{g.f.} = -\frac{1}{2\xi}|\partial^{\mu}W^{-}_{\mu} - \xi m_W w^-|^2 - \frac{1}{2\xi}|\partial^{\mu}W^{+}_{\mu} - \xi m_W w^+|^2 \tag{3}$$

$$-\frac{1}{2\eta}(\partial^{\mu}Z_{\mu} - \eta m_Z z)^2 - \frac{1}{2\alpha}(\partial^{\mu}A_{\mu})^2$$

The meaning of the choice (3) is that the quadratic part of the Lagrangian then becomes diagonal (a mixing of the $w^{\pm}$ and $W^{\pm}$ etc. is removed). Indeed, by substituting (2) into the original gauge invariant SM Lagrangian one gets, in particular

$$(D_{\mu}\Phi)^{\dagger}(D^{\mu}\Phi) = m_W(\partial^{\mu}w^- W^+_{\mu} + \partial^{\mu}w^+ W^-_{\mu}) + m_Z \partial^{\mu}z Z_{\mu} + \ldots \tag{4}$$

In (4) we have written explicitly only the above-mentioned mixing terms involving vector bosons and their unphysical scalar partners. On the other hand, the gauge-fixing term (3) also produces some bilinear terms of the above-mentioned type, namely

$$\mathcal{L}_{g.f.} = m_W(w^-\partial^{\mu}W^+_{\mu} + w^+\partial^{\mu}W^-_{\mu}) + m_Z z\partial^{\mu}Z_{\mu} + \ldots \tag{5}$$

Adding (4) and (5) one gets

$$\mathcal{L}_{Higgs} + \mathcal{L}_{g.f.} = m_W\partial^{\mu}(w^- W^+_{\mu} + w^+ W^-_{\mu}) + m_Z\partial^{\mu}(zZ_{\mu}) + \ldots \tag{6}$$

i.e. the bilinear terms combine into four-divergences and may therefore be omitted as we have already indicated above.

Let us now examine the remaining quadratic terms involving unphysical scalars. From $\mathcal{L}_{Higgs}$ (see (A.28)) one gets readily kinetic terms for the $w^\pm$ and z and $\mathcal{L}_{g.f.}$ yields the corresponding "mass terms". Taken together, these terms amount to

$$\mathcal{L}_{Higgs} + \mathcal{L}_{g.f.} = \partial^\mu w^- \partial_\mu w^+ + \frac{1}{2}\partial^\mu z \partial_\mu z - \xi m_W^2 w^- w^+ - \frac{1}{2}\eta m_Z^2 z^2 + \ldots \quad (7)$$

From (7) one may read off the mass parameters

$$m_w^2 = \xi m_W^2, \quad m_z^2 = \eta m_Z^2 \quad (8)$$

These expressions exhibit a dependence of the $w^\pm$ and z "masses" on the (arbitrary) gauge parameters ξ, η; this reflects the unphysical nature of the would-be Goldstone bosons which actually disappear from the physical spectrum. On the other hand, let us emphasize that the mass term for the physical Higgs boson H comes from the potential (A.30) upon the shift involved in (2) and thus it is of course gauge-independent (one has $m_H^2 = 2\lambda v^2$).

Quadratic terms involving vector boson fields descend from $\mathcal{L}_{gauge}$ (kinetic terms — see (A.25)), from $\mathcal{L}_{Higgs}$ (mass terms for W and Z) and from $\mathcal{L}_{g.f.}$. One may summarize them as

$$\begin{aligned}\mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{g.f.} = & -\frac{1}{2}W^-_{\mu\nu}W^{+\mu\nu} - \frac{1}{\xi}(\partial\cdot W^-)(\partial\cdot W^+) + m_W^2 W^-_\mu W^{+\mu} \\ & -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{2\eta}(\partial\cdot Z)^2 + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \quad (9) \\ & -\frac{1}{4}A_{\mu\nu}A^{\mu\nu} - \frac{1}{2\alpha}(\partial\cdot A)^2 + \ldots\end{aligned}$$

In quantized theory, propagators of vector bosons are obtained by inverting the quadratic form in (9) (for a standard technique of doing so, see e.g. Appendix H in [7]). The result is

$$\begin{aligned}D^{(W)}_{\mu\nu}(k) &= [-g_{\mu\nu} + (1-\xi)(k^2 - \xi m_W^2)^{-1}k_\mu k_\nu]\frac{1}{k^2 - m_W^2 + i\varepsilon} \\ D^{(Z)}_{\mu\nu}(k) &= [-g_{\mu\nu} + (1-\eta)(k^2 - \eta m_W^2)^{-1}k_\mu k_\nu]\frac{1}{k^2 - m_Z^2 + i\varepsilon} \quad (10)\end{aligned}$$

$$D^{(A)}_{\mu\nu}(k) = [-g_{\mu\nu} + (1-\alpha)(k^2)^{-1}k_\mu k_\nu]\frac{1}{k^2 + i\varepsilon}$$

The expressions (10) are seen to behave like k^{-2} for $k \to \infty$ and this explains the term "renormalizable gauges". Let us stress again that such a decent ultraviolet behaviour of the massive vector boson propagators has been achieved at the price of introducing unphysical degrees of freedom — the "Goldstone scalar ghosts" $w^\pm$ and z. Let us also add that for a general R-gauge (3) the propagators of $w^\pm$ and z are, in view of (7)

$$D^{(w)}(k) = \frac{1}{k^2 - \xi m_W^2 + i\varepsilon} \tag{11}$$

$$D^{(z)}(k) = \frac{1}{k^2 - \eta m_Z^2 + i\varepsilon}$$

From (10) it is obvious that for practical calculations, the most convenient choice of gauge corresponds to $\xi = \eta = \alpha = 1$ as the vector boson propagators are then diagonal; this is the familiar 't Hooft – Feynman gauge. Note that in this gauge the (unphysical) mass parameters of the scalars $w^\pm$, z are equal to the masses of their vector boson counterparts $W^\pm$, Z. One may also notice that taking the limit $\xi \to \infty$, $\eta \to \infty$ in (10) one recovers the U-gauge propagators of W and Z (cf. (11)). Furthermore, the expressions (11) are seen to vanish identically in such a limit (the $w^\pm$, z become "infinitely heavy"); this is of course gratifying since the $w^\pm$ and z should be absent in the U-gauge (by definition).

It is clear that the SM interaction Lagrangian in an R-gauge will contain many additional contributions in comparison with the U-gauge case since now one must also consider terms involving the unphysical scalars. A complete catalogue of the R-gauge interaction vertices may be found in many places (see e.g. [25], [26]). Here we will restrict ourselves only to some instructive examples.

First, when (2) is used in $\mathcal{L}_{Higgs}$, the expression $(D^\mu\Phi)^\dagger(D_\mu\Phi)$ yields, among other things, a trilinear interaction involving two scalars $w^\pm$ and a (neutral) vector boson, which reads

$$\mathcal{L}_{w^-w^+V} = iew^-\overleftrightarrow{\partial^\mu}w^+A_\mu + i\frac{g}{\cos\vartheta_W}(\frac{1}{2} - \sin^2\vartheta_W)w^-\overleftrightarrow{\partial^\mu}w^+Z_\mu \tag{12}$$

In this context let us remark that many conceivable types of R-gauge interaction terms may be formally deduced from U-gauge vertices by replacing one or more vector boson lines by the corresponding unphysical Goldstone bosons. The expression (12) is an explicit example of such an interaction term.

Second, considering the $\mathcal{L}_{Yukawa}$ for a lepton l (see (A.41)) one gets, using (2)

$$\mathcal{L}_{Yukawa} = i\frac{g}{2\sqrt{2}}\frac{m_l}{m_W}\overline{\nu}(1+\gamma_5)lw^+ + \text{h.c.} \tag{13}$$

$$-i\frac{g}{2}\frac{m_l}{m_W}\bar{l}\gamma_5 lz - \frac{g}{2}\frac{m_l}{m_W}\bar{l}lH$$

(the standard lepton–Higgs interaction is of course recovered in (13) as expected).

Third, there are new scalar self-interactions descending from the "potential" $V(\Phi)$ in $\mathcal{L}_{Higgs}$. One has

$$\mathcal{L}_{\text{int.}}^{(\text{scalar})} = -\lambda vH(2w^-w^+ + z^2 + H^2) - \frac{1}{4}\lambda(2w^-w^+ + z^2 + H^2)^2 \tag{14}$$

Again, (14) incorporates also the Higgs boson self-interactions known from the U-gauge formulation.

All this, however, is not the whole story yet. The standard model is a non-abelian gauge theory and when it is quantized in an R-gauge (3), one has to introduce another set of unphysical fields, namely the Faddeev–Popov (FP) ghosts (which do not occur in the U-gauge). Note that an essential reason for invoking the FP ghosts in a gauge like (3) is that otherwise the S-matrix would not be unitary at one-loop level (see e.g. [26], [27]). Thus, a complete relevant Lagrangian in the considered R-gauge reads, schematically

$$\mathcal{L}_{SM}^{(R-gauge)} = \mathcal{L}_{g.inv.} + \mathcal{L}_{g.f.} + \mathcal{L}_{FP} \tag{15}$$

The FP term can be derived most efficiently by means of the path-integral techniques (see [25]). A detailed form of the $\mathcal{L}_{FP}$ will not be needed in what follows, so we do not reproduce it here. For the purpose of later references we will only summarize briefly some essential features of the quantum R-gauge SM Lagrangian (15).

First let us introduce a convenient shorthand notation for the gauge-fixing (GF) functions occurring in (3), namely

$$F_a[V,\varphi] = \begin{cases} \partial^\mu W^\pm_\mu - \xi m_W w^\pm \\ \partial^\mu Z_\mu - \eta m_Z z \\ \partial^\mu A_\mu \end{cases} \tag{16}$$

(on the l.h.s. of (16), the index a labels the four SM vector fields denoted collectively by V, and φ stands for the unphysical Higgs-Goldstone fields).

The FP term in (15) involves four ghost fields c_a (associated with the four gauge bosons) and the corresponding conjugate (antighost) variables $\bar{c}_a$. The structure of $\mathcal{L}_{FP}$ is determined by the gauge variation of the GF functions F_a (see [25]). FP ghosts represent (unphysical) Lorentz scalars obeying Fermi statistics and enter Feynman diagrams only via closed loops. Let us remark that the corresponding mass-squared parameters are ξm_W^2 for $c_\pm$, ηm_Z^2 for c_Z and 0 for c_γ. Further, the interaction terms contained in $\mathcal{L}_{FP}$ are such that a pair of FP ghosts (being not both neutral) is coupled to another field which may be a vector ($W^\pm, Z$ or γ), the Higgs boson H, or an unphysical Goldstone scalar ($w^\pm$ or z).

The most remarkable property of the full SM Lagrangian (15) is a peculiar *global* symmetry discovered by Becchi, Rouet and Stora [28] (and independently by Tyutin [29]) which represents, in a sense, a "remnant" of the original classical gauge symmetry, broken by the gauge-fixing procedure, i.e. by including the $\mathcal{L}_{g.f.}$ and the associated term $\mathcal{L}_{FP}$. Such a "residual" symmetry is in fact a general feature of quantized gauge theories (for a review, see [30]) and nowadays it is an issue discussed in most textbooks on modern field theory (see e.g. [25], [26], [27]). In the present context, the BRS transformations may be written (rather schematically) as

$$\begin{aligned} \delta V_a^\mu &= \theta D_{ab}^\mu c_b \\ \delta\Psi &= -i\theta T_a c_a \Psi \\ \delta\bar{c}_a &= -\theta\frac{1}{\xi}F_a \\ \delta c_a &= -\theta\frac{1}{2}f_{abd}c_b c_d \end{aligned} \tag{17}$$

where D^μ is the relevant covariant derivative (in the adjoint representation), Ψ is a generic symbol for the matter fields, T_a is a gauge group generator,

f_{abd} denotes a corresponding structure constant, and F_a is given by (16). For simplicity, all gauge-fixing parameters are denoted by ξ and we have also suppressed the coupling constants. The parameter θ is a constant *anticommuting* (Grassmann) real number; it means that $\theta^2 = 0$ and θ is thus effectively infinitesimal.

Now it is well known (see e.g. [26], [27]) that by using the BRS symmetry one can recover the Ward-Takahashi (Slavnov-Taylor) identities of a quantized gauge theory. Thus, the global BRS symmetry describes concisely the contents of the local gauge invariance at quantum level. Quantization of non-abelian gauge theories is usually implemented in the path-integral formalism. However, as we shall see later, in some situations it may be useful to have at hand also a covariant canonical operator method. Such a quantization procedure (which, roughly speaking, is a generalization of the well-known Gupta-Bleuler method in QED) was indeed invented by Kugo and Ojima [31] (see also [32], [33] and references therein). In their approach, the notion of BRS symmetry plays a crucial role. The main breakthrough of [31] consists in a successful generalization of the Gupta-Bleuler subsidiary condition, characterizing a subspace of physical states. Let us recall that the GB condition may be written as

$$\partial^\mu A_\mu^{(-)}(x)|\psi_{\text{phys.}}>= 0 \tag{18}$$

where $A_\mu^{(-)}(x)$ denotes the annihilation part of quantized electromagnetic potential. According to [31], a correct generalization of (18) to the non-abelian case is astonishingly simple, yet highly non-trivial; it reads

$$Q_{BRS}|\psi_{\text{phys.}}>= 0 \tag{19}$$

where Q_{BRS} is an operator version of the conserved Noether charge corresponding to BRS symmetry (note that for an abelian gauge field it can be shown that (19) implies (18), so (19) may indeed be viewed as a natural extension of (18) in a general case). The canonical (anti)commutation relations set up in [31] ensure that Q_{BRS} is the generator of transformations (17) for the corresponding field operators (let us emphasize that the FP ghosts satisfy anticommutation relations). An example of such an operator relation, which will be useful in subsequent discussion, is provided by the anticommutator

$$\{Q_{BRS}, \bar{c}_a\} = -\frac{1}{\xi}F_a \tag{20}$$

(note that (20) corresponds to the BRS transformation of the antighost field in (17)). Let us also add that the BRS charge has another remarkable property: It is *nilpotent*, i.e.

$$Q_{BRS}^2 = 0 \tag{21}$$

(it follows from (21) that BRS transformations of state vectors and operators are effectively infinitesimal).

In the section 4 we will invoke only some basic ingredients of the Kugo-Ojima canonical operator quantization scheme mentioned above, at the level necessary for understanding a basic idea of the proof of Equivalence Theorem for longitudinal vector bosons.

Exercise: *At the tree level prove that the scattering amplitudes for* $e^+e^- \longrightarrow \mu^+\mu^-$ *in a R-gauge and in U-gauge are equal.*

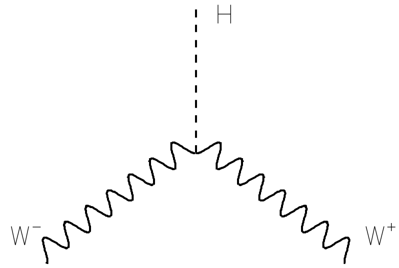
3 Equivalence Theorem – examples

A familiar part of the physical "folklore" is a rather vague but frequently used statement concerning the Higgs mechanism, which may be paraphrased roughly as follows:

The would-be Goldstone bosons are "eaten" by the gauge fields which become massive and the massive vector bosons may have — in contrast to massless ones — also longitudinal polarizations. Thus, in a sense, the unphysical Goldstone scalars become the longitudinal components of massive vector bosons.

It is remarkable that such an intuitive statement may be given a more precise meaning on the level of S-matrix elements. Indeed, this is the contents of the Equivalence Theorem [8] – [15], stating that in high-energy limit, the S-matrix element for a process with external longitudinally polarized vector bosons is equal, up to a constant factor, to a matrix element (calculated within R-gauge) in which longitudinal vector bosons are replaced by the corresponding unphysical Higgs-Goldstone scalars. We will formulate the theorem explicitly later in this section; now we are going to give two instructive examples of how such an equivalence works "in practice".

As a first example let us consider the decay of a very heavy Higgs boson ($m_H \gg 2m_W$) into a pair of longitudinally polarized vector bosons $W^\pm$. In lowest order, the process is described by the diagram



and the corresponding Lorentz invariant matrix element may be written as (cf. A.38))

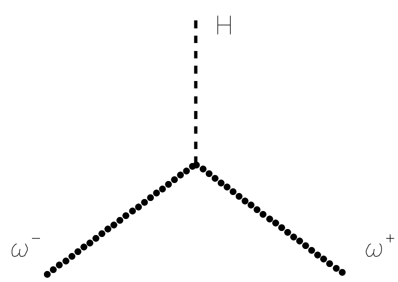
$$\mathcal{M}(H \to W_L^- W_L^+) = g m_W \varepsilon_L^\mu(k) \varepsilon_{L\mu}(p) \tag{22}$$

$$= g m_W (m_W^{-1} k^\mu + \Delta^\mu(k))(m_W^{-1} p_\mu + \Delta_\mu(p))$$

In (22) we have used the well-known high-energy decomposition of longitudinal polarization vectors (for details see e.g. Appendix H in [7]); note that the quantities $\Delta^\mu(k)$, $\Delta^\mu(p)$ are of an order of $\mathcal{O}(m_W/E_W)$ where of course $E_W = \frac{1}{2} m_H$, i.e. $\Delta^\mu \ll 1$ in our case. Thus, from (22) one gets, after some simple manipulations

$$\mathcal{M}(H \to W_L^- W_L^+) = \frac{g}{2} \frac{m_H^2}{m_W} (1 + \mathcal{O}(m_W^2/E_W^2)) \tag{23}$$

On the other hand, for the unphysical process $H \to w^- w^+$ described formally within an R-gauge by the lowest-order diagram



one gets the matrix element (see (14))

$$\mathcal{M}(H \to w^- w^+) = -\frac{g}{2}\frac{M_H^2}{m_W} \tag{24}$$

Comparing (23) and (24) one may observe that these two matrix elements are indeed equal, up to a correction suppressed by a factor of m_W^2/E_W^2 and up to a minus sign.

As another, less trivial example let us consider the process $e^+e^- \to W_L^- W_L^+$. For simplicity, we will assume $m_e = 0$. The lowest-order (i.e. tree-level) scattering amplitude for such a process is a sum of several Feynman diagrams (as an exercise, draw these diagrams in U-gauge and in R-gauge resp.) and one gets, after a rather long calculation

$$\mathcal{M}(e^+e^- \to W_L^- W_L^+) = \frac{1}{s}\overline{v}(k_2)(\not{p}_1 - \not{p}_2)u(k_1)\times \tag{25}$$

$$\times[e^2 + g^2(-\frac{1}{2} + \sin^2\vartheta_W)(\frac{m_Z^2}{2m_W^2} - 1)] + \mathcal{O}(m_W^2/s)$$

where k_1, k_2, p_1 and p_2 are consecutively the four-momenta of e^-, e^+, W^- and W^+ and s is the usual Mandelstam invariant. On the other hand, for the unphysical process $e^+e^- \to w^-w^+$ one gets (e.g. in the 't Hooft – Feynman gauge)

$$\mathcal{M}(e^+e^- \to w^- w^+) = -\frac{1}{s}\overline{v}(k_2)(\not{p}_1 - \not{p}_2)u(k_1)\times \tag{26}$$

$$\times[e^2 + (\frac{g}{\cos\vartheta_W})^2(\frac{1}{2} - \sin^2\vartheta_W)^2] + \mathcal{O}(m_W^2/s)$$

Note that the matrix element (26) is again obtained by summing several tree-level Feynman diagrams (which graphs do actually contribute ?); the rules for the relevant vertices follow from (12) and (13). Now, using in (26) the relation $m_W/m_Z = \cos\vartheta_W$, the expressions (25) and (26) are seen to coincide, up to a sign and barring the corrections suppressed by $m_W^2/s \ll 1$.

The above two examples illustrate the general theorem mentioned earlier in this section, namely

Equivalence Theorem: Let us consider a process involving, apart from other physical particles, a certain number of longitudinally polarized vector bosons V_L (i.e. $W_L^{\pm}$ and/or Z_L), with n_1 of them being in the initial state and n_2 in the final state. Let E_V denote generically the vector-boson energies; for $E_V \gg m_V$ one then has

$$\begin{aligned} & \mathcal{M}_{fi}(V_L(i_1), \ldots, V_L(i_{n_1}), A \to V_L(f_1), \ldots, V_L(f_{n_2}), B) = \\ = \; & \mathcal{M}_{fi}(\varphi(i_1), \ldots, \varphi(i_{n_1}), A \to \varphi(f_1), \ldots, \varphi(f_{n_2}), B) \times \\ \times \; & i^{n_1}(-i)^{n_2} C[1 + O(m_V/E_V)] \end{aligned} \tag{27}$$

where the φ's stand for the unphysical Higgs-Goldstone scalar counterparts of the V_L's, the A, B symbolize all other incoming and outgoing particles and the factor C is in general a constant independent of the energies which is due to renormalization effects (C is a product of factors associated with individual external Higgs-Goldstone fields, $C = 1$ at the tree level). ♠

Note that the phase factor contained in (27) is indeed recovered in our two previous examples (where $n_1 = 0, n_2 = 2$). The idea of a general proof of ET is the subject of the next section (except the discussion of the C, for which the reader is referred to the original literature [12] – [15]).

Exercise: *Verify the validity of ET (27) at the tree level in the case of decay $l \to W + \nu_l$, where l is a hypothetical heavy lepton ($m_l \gg m_W$) with usual SM couplings.*

4 Idea of ET proof – a basic Ward identity

While the explicit examples given in preceding section clearly support the validity of ET (27), one would like to know what is actually the true origin of such a remarkable statement. In this section we will outline briefly the idea of a formal proof of ET, following essentially the work of Gounaris et al. [11].

Intuitively, a connection between longitudinal vector bosons and unphysical Higgs-Goldstone scalars is suggested by the form of the GF functions (16), and one may expect that ET should follow formally from an appropriate Ward identity reflecting the gauge invariance of the considered electroweak theory. This is indeed the case. The foundation for a formal proof of ET is provided by the identity

$$< B|T[F_{a_1}(x_1) \ldots F_{a_n}(x_n)]|A >_{con} = 0 \tag{28}$$

where $F_a(x)$ is given by (16) and $|A>$, $|B>$ are *physical* in- and out-states (i.e. they satisfy the condition (19)). The subscript at the matrix element denotes its "connected part": It means that the terms, which correspond to factorization of a vacuum-to-vacuum matrix element involving the F_a's, are discarded (it is shown in [11] that such terms have contact character - they are proportional to products of the delta-functions $\delta^4(x_j - x_k)$). First we are going to discuss the origin of (28) and its connection with ET will be explained later. We are using the framework of canonical operator quantization of non-abelian gauge theories established in [31], [32], [33] (note that the treatment of ref. [11] does not rely explicitly on the operator formalism).

One may start with the following observation: For a matrix element of a string of local operators between physical states (characterized by (19)) one gets the identity

$$0 = \sum_{k-1}^{n} < B|T[O_1(x_1) \ldots O_{k-1}(x_{k-1}) \delta_{BRS} O_k(x_k) O_{k+1}(x_{k+1}) \ldots O_n(x_n)]|A > \tag{29}$$

where the (infinitesimal) BRS transformation of an operator O_k is given by an (anti)commutator with the generator Q_{BRS} (cf. the discussion around eq. (20)). Such a relation is obtained immediately, if one considers an (anti)commutator of Q_{BRS} with the relevant operator product, sandwiched between the physical states: On the one hand, this obviously vanishes owing to (19); on the other hand, working it out one gets the sum on the r.h.s. of (29). (Note that since (29) emerges as a consequence of the BRS symmetry of the underlying theory, it represents the prototype of a general Ward identity.) The identity (28) can now be proved by utilizing a set of the relations (29) with operators $O_k(x)$ being either the antighost fields $\bar{c}_a(x)$ or the GF functions $F_a(x)$. Starting with $n = 1$, one may choose $O_1(x) = \bar{c}_a(x)$ in (29); then using (20) it follows immediately

$$< B|F_a(x)|A >= 0 \tag{30}$$

In fact, it is not difficult to realize that this already proves (28) for $n = 1$ (in (30) there cannot be any "disconnected" term of the above-mentioned type – this is obvious e.g. from the explicit form of the F_a as given by (16)). For $n = 2$, one chooses

$$O_1(x_1) = F_{a_1}(x_1), \qquad O_2(x_2) = \bar{c_{a_2}}(x_2) \tag{31}$$

in (29). In this case, one must invoke an additional fact about BRS symmetry which has not been mentioned so far, namely that the BRS transform of a gauge-fixing function is proportional to the equation of motion of the associated antighost (see e.g. [30]). Equipped with this knowledge, and using eq. (20) as well, one arrives at the identity (28) with $n = 2$ (after discarding a disconnected contact term, proportional to $\delta^4(x_1 - x_2)$). One may proceed further in this way following (31), and complete the proof by induction. Here we have only sketched its basic idea; for more technical details the reader is referred to [11]. Let us add that a virtue of the operator formalism in the present context consists mainly in the compact characterization of physical states by means of the condition (19).

We will now show how ET follows from the identity (28). To demonstrate a basic idea of the proof, we are going to discuss in detail first the simplest case involving a single longitudinal vector boson. Later on we will indicate, by means of a particular example, how the procedure can be generalized. Thus, let us consider a process $A \to B + V_a$, where V_a stands for $W^\pm$ or Z and A, B denote other physical particles (including possibly other vector bosons as well). The corresponding amplitude, i.e. an S-matrix element, can be expressed by means of an appropriate reduction formula (obviously, such a representation is useful in view of an envisaged application of (28)). For a "truncated matrix element" of the indicated process one may then write, schematically

$$\mathcal{M}_\mu(A \to B + V_a(p,\lambda)) = \mathrm{FT}\{L_{\mu\nu} < B|V_a^\nu(x)|A >\} \tag{32}$$

Here "truncation" means removing the polarization vector of the V_a from the full matrix element $\mathcal{M}$, expressed as $\mathcal{M} = \mathcal{M}_\mu \varepsilon^\mu(p,\lambda)$. The symbol FT in (32) denotes Fourier transformation and $L_{\mu\nu}$ is the linear differential operator which appears in the equation of motion for V_a; it reads

$$L_{\mu\nu} = (\Box + m_a^2)g_{\mu\nu} + (\frac{1}{\xi} - 1)\partial_\mu\partial_\nu \tag{33}$$

(note that (33) corresponds to the quadratic part of the Lagrangian (9)). Let us also remark that we suppress systematically factors related to normalization of one-particle states, or those which are due to conventions adopted in defining the considered matrix elements. This is justified since our result will be a simple linear relation between two matrix elements (with a similar

structure) in which all such extra factors cancel. From (32) one gets further

$$ip^{\mu}\mathcal{M}_{\mu} = \mathrm{FT}\left\{\partial^{\mu}L_{\mu\nu}\left\langle B\left|V_{a}^{\nu}\left(x\right)\right|A\right\rangle\right\} \tag{34}$$

Taking into account (33), this becomes

$$ip^{\mu}\mathcal{M}_{\mu} = \mathrm{FT}\left\{\left(m_a^2+\frac{1}{\xi}\Box\right)\left\langle B\left|\partial_{\nu}V_{a}^{\nu}\left(x\right)\right|A\right\rangle\right\} \tag{35}$$

According to (16), $\partial_{\nu}V_{a}^{\nu}\left(x\right)$ may be recast as $F_a+\xi m_a\varphi_a$, so in the r.h.s. of (35) one may employ subsequently the identity (28) with $n=1$ (i.e. eq.(30)). One thus gets

$$i\frac{p^{\mu}}{m_a}\mathcal{M}_{\mu} = \mathrm{FT}\left\{\left(\Box+\xi m_a^2\right)\left\langle B\left|\varphi_a\left(x\right)\right|A\right\rangle\right\} \tag{36}$$

However, the expression on the r.h.s. of (36) gives (via a reduction formula) just the amplitude of the process $A\longrightarrow B+\varphi_a\left(p\right)$. Thus, we arrive at the identity

$$i\frac{p^{\mu}}{m_a}\mathcal{M}_{\mu}\left(A\longrightarrow B+V_a\left(p,\lambda\right)\right) = \mathcal{M}\left(A\longrightarrow B+\varphi_a\left(p\right)\right) \tag{37}$$

It should be noted that for a corresponding process with an *incoming* vector boson (carrying four-momentum p) one would obtain an analogous relation with opposite sign in the left-hand side, i.e. with $i\rightarrow -i$. Let us emphasize that up to now the polarization of the vector boson was completely arbitrary, i.e. (37) is valid for any λ. Consider now the case $\lambda=L$ and assume also that the states A, B do not contain any other longitudinal vector boson. The vector of longitudinal polarization can be decomposed as

$$\varepsilon_L^{\mu}\left(p\right) = \frac{1}{m_a}p^{\mu}+\Delta^{\mu}\left(p\right) \tag{38}$$

(c.f. (22) etc.) where the $\Delta^{\mu}\left(p\right)$ behaves as $\mathcal{O}\left(m_a/E\right)$ for $E\gg m_a$. Taking into account (38), one then gets immediately

$$\mathcal{M}_{\mu}\varepsilon_L^{\mu}\left(p\right) = \frac{1}{m_a}p^{\mu}\mathcal{M}_{\mu}+\mathcal{O}\left(\frac{m_a}{E}\right) \tag{39}$$

in the high-energy limit; using (37), it can be further rewritten as

$$\mathcal{M}\left(A \longrightarrow B+V_a\left(p,\lambda=L\right)\right)=-i\mathcal{M}\left(A \longrightarrow B+\varphi_a\left(p\right)\right)\left[1+\mathcal{O}\left(\frac{m_a}{E}\right)\right] \tag{40}$$

Note that in writing (39) and (40) we have also tacitly assumed that both $\mathcal{M}_\mu$ and $\mathcal{M}\left(A \longrightarrow B+\varphi_a\right)$ are bounded in the high-energy limit; this is justified as these quantities do not contain any source of a "bad" high-energy behaviour. The relation (40) is just the statement of the "naive" ET for a single longitudinal vector boson in the final state (cf. (27) with $n_1=0, n_2=1$ and $C=1$). Of course, for a process with one longitudinal vector boson in the initial state one would obtain an analogous relation, with the $-i$ being replaced by i (cf. the remark following eq.(37)). The term "naive" refers to the fact that throughout our discussion we have neglected the relevant renormalization effects which may in general lead to the finite modification factors incorporated in eq. (27). A detailed discussion of a such an issue would go beyond the scope of this introductory review and the interested reader is referred to the original papers [12] – [15]. Let us also remark that, according to the analysis performed in [15], the modification factor C in (27) can be made equal to unity in a suitable renormalization scheme. Anyway, even in the general case with $C\neq 1$ a finite multiplicative constant does not bear on the issue of the asymptotic behaviour of scattering amplitudes in SM for energies much larger than any mass involved in the theory.

Having demonstrated the basic idea of ET proof, we are not going to discuss the general case in detail. Instead, we will only indicate how one could proceed further: Next we are going to formulate the relevant generalization of the identity (37) and show subsequently how it can be employed, by analysing a particular example. To this end, let us first introduce a useful notation due to Chanowitz and Gaillard [10]. Following [10], one may define five-component objects $D^a_M\left(a\right)$ and $\mathcal{M}_M$ with $M=0,1,2,3,4$ as

$$\begin{aligned} D^a_M\left(p\right) &= \left(-ip_\mu, m_a\right) \\ \mathcal{M}_M &= \left(\mathcal{M}_\mu, \mathcal{M}_4\right) \end{aligned} \tag{41}$$

where $\mathcal{M}_\mu$ has the usual meaning and $\mathcal{M}_4=\mathcal{M}\left(A \longrightarrow B+\varphi_a\right)$; we have thus introduced formally the (trivial) "truncation" of a matrix element for emission of a scalar φ_a. Note that we do not distinguish between $\mathcal{M}_4$ and $\mathcal{M}^4$ etc. In terms of the symbols (41), the identity (37) may be written

compactly as

$$D_M^a(p)\,\mathcal{M}^M = 0 \tag{42}$$

The generalization of eq.(37) (or, equivalently, (42)) for matrix elements involving an arbitrary number of vector bosons and/or unphysical Higgs-Goldstone scalars corresponds to a general n in the basic Ward identity (28). For simplicity, let us restrict ourselves to the case where all vector bosons are outgoing. The relevant identity obtained first in [10] reads

$$D_{M_1}^{a_1}(p_1)\cdots D_{M_n}^{a_n}(p_n)\,\mathcal{M}^{M_1\ldots M_n} = 0 \tag{43}$$

where $\mathcal{M}^{M_1\ldots M_n}$ denotes a (partially) truncated matrix element, in the sense specified above. By "partial truncation" we mean that such a matrix element may still incorporate polarization vectors of additional vector bosons (contained in the physical states A, B in (28)). Note also that in the most general case involving both outgoing and incoming vector bosons, the extension of eq.(43) includes also the conjugate "five-vectors" $\widetilde{D}_M$ given by

$$\widetilde{D}_M^a(p) = (ip_\mu, m_a) \tag{44}$$

which correspond just to the incoming vector bosons and unphysical scalars (cf. the remark following eq.(37)).

Chanowitz and Gaillard [10] employed the set of identities (43) to accomplish a general proof of ET. Here we will only illustrate how eq.(43) can be utilized in such an analysis, by considering a particular example, namely the process $e^+e^- \longrightarrow W_L^- W_L^+$ (which we have discussed at an elementary level in preceding section — cf.(26), (26)). To begin with, let us introduce the truncated matrix elements associated with the considered process; these are defined by

$$\begin{aligned}
\mathcal{M}\left(e^-e^+ \longrightarrow W^-W^+\right) &= \mathcal{M}_{\mu\nu}\varepsilon^\mu(p_1)\,\varepsilon^\nu(p_2)\\
\mathcal{M}\left(e^-e^+ \longrightarrow w^-W^+\right) &= \mathcal{M}_{4\nu}\varepsilon^\nu(p_2)\\
\mathcal{M}\left(e^-e^+ \longrightarrow W^-w^+\right) &= \mathcal{M}_{\mu4}\varepsilon^\mu(p_1)\\
\mathcal{M}\left(e^-e^+ \longrightarrow w^-w^+\right) &= \mathcal{M}_{44}
\end{aligned} \tag{45}$$

(we denote the relevant four-momenta as in (25), (26)). In the subsequent discussion we will employ the identities (43) with $n = 1$ and $n = 2$, namely

$$\begin{aligned}
D^M(p_1)\,\mathcal{M}_{M\nu}\varepsilon^\nu(p_2) &= 0\\
D^N(p_2)\,\mathcal{M}_{\mu N}\varepsilon^\mu(p_1) &= 0
\end{aligned} \tag{46}$$

and

$$D^M (p_1) D^N (p_2) \mathcal{M}_{MN} = 0 \tag{47}$$

(to simplify the notation, we have suppressed here the labels a_j). Using the definition (41), the relations (46) and (47) may be worked out as

$$\begin{aligned} \left(-ip_1^\mu \mathcal{M}_{\mu\nu} + m_W \mathcal{M}_{4\nu}\right) \varepsilon^\nu (p_2) &= 0 \\ \left(-ip_2^\nu \mathcal{M}_{\mu\nu} + m_W \mathcal{M}_{\mu 4}\right) \varepsilon^\mu (p_1) &= 0 \end{aligned} \tag{48}$$

and

$$-p_1^\mu p_2^\nu \mathcal{M}_{\mu\nu} - ip_1^\mu m_W \mathcal{M}_{\mu 4} - ip_2^\nu m_W \mathcal{M}_{4\nu} + m_W^2 \mathcal{M}_{44} = 0 \tag{49}$$

(up to now, vector boson polarizations may be arbitrary). Let us now show that ET is valid for the considered process. Denoting the amplitude for $e^- e^+ \longrightarrow W_L^- W_L^+$ simply by $\mathcal{M}_{LL}$ and employing the decomposition (38) for longitudinal polarization vectors, we get first

$$\begin{aligned} \mathcal{M}_{LL} &= \mathcal{M}_{\mu\nu} \varepsilon_L^\mu (p_1) \varepsilon_L^\nu (p_2) \qquad (50) \\ &= \left[\frac{1}{m_W^2} p_1^\mu p_2^\nu + \frac{1}{m_W} p_1^\mu \Delta^\nu (p_2) + \frac{1}{m_W} \Delta^\mu (p_1) p_2^\nu + \Delta^\mu (p_1) \Delta^\nu (p_2)\right] \mathcal{M}_{\mu\nu} \end{aligned}$$

The quantities $\Delta (p)$ occurring in the second and the third term of (50) can be re-expressed by means of (38); after a simple manipulation we thus obtain

$$\mathcal{M}_{LL} = \left[-\frac{1}{m_W^2} p_1^\mu p_2^\nu + \frac{1}{m_W} p_1^\mu \varepsilon_L^\nu (p_2) + \frac{1}{m_W} \varepsilon_L^\mu (p_1) p_2^\nu + \Delta^\mu (p_1) \Delta^\nu (p_2)\right] \mathcal{M}_{\mu\nu} \tag{51}$$

Employing now the identities (48) and (49), the expression (51) becomes

$$\begin{aligned} \mathcal{M}_{LL} &= i\frac{p_1^\mu}{m_W} \mathcal{M}_{\mu 4} + i\frac{p_2^\nu}{m_W} \mathcal{M}_{4\nu} - \mathcal{M}_{44} \\ &\quad -i\varepsilon_L^\mu (p_1) \mathcal{M}_{\mu 4} - i\varepsilon_L^\nu (p_2) \mathcal{M}_{4\nu} + \Delta^\mu (p_1) \Delta^\nu (p_2) \mathcal{M}_{\mu\nu} \qquad (52) \\ &= -\mathcal{M}_{44} - i\Delta^\mu (p_1) \mathcal{M}_{\mu 4} - i\Delta^\nu (p_2) \mathcal{M}_{4\nu} + \Delta^\mu (p_1) \Delta^\nu (p_2) \mathcal{M}_{\mu\nu} \end{aligned}$$

For $E \gg m_W$, where E is the W boson energy, the quantities $\Delta (p)$ are of the order $\mathcal{O} (m_W/E)$ and the truncated matrix elements $\mathcal{M}_{MN}$ in (52) are at most $\mathcal{O} (1)$. One may thus finally write, in the high-energy limit

$$\mathcal{M}_{LL} = -\mathcal{M}_{44} \left[1 + \mathcal{O} (m_W/E)\right] \tag{53}$$

which is just the statement of the ET for the considered particular case, including the phase factor (-1) (cf.(27) with $n_1 = 0$, $n_2 = 2$).

Let us add that the result (52) can be generalized in a straightforward way for processes involving an arbitrary number of external longitudinal vector bosons. A corresponding relation (which is implicit in the treatment [10]) has been formulated explicitly first by H.Veltman (see ref.[20]). Recently, it has been discussed by Grosse-Knetter and Kuss [16] who refer to an identity of the type (52) as the *generalized equivalence theorem.* Following [16], it should be emphasized that eq.(52) is an *exact* relation and the usual ET form of the type (53) is obtained only when the truncated matrix elements in (52) exhibit a "soft" high-energy behaviour (which is obvious in R-gauge formulation of the SM assuming that Higgs boson H is not much heavier than the W.)

5 Conclusion

Concluding these notes, let us return briefly to the problem of high-energy behaviour of Feynman graphs within SM as outlined in the Introduction. It is now clear that with ET at hand, the tree unitarity [3], [7] (i.e. the "asymptotic softness" [6] of tree-level amplitudes) can be proved in two steps: First, one may invoke gauge-independence of S-matrix elements, which is a consequence of Ward identities of the theory (see e.g. [25] for a proof within the class of R_ξ-gauges and [34] for a discussion of the U-gauge limit). By passing from U-gauge to an R-gauge, one gets rid of a potential source of "bad" high-energy behaviour, residing in the U-gauge vector boson propagators (which contain pieces proportional to m_V^{-2}).

Second, having passed to R-gauge, one may employ ET (which also originates in a particular Ward identity) and this in turn eliminates another possible source of troubles, namely the vectors of longitudinal polarization (which contain pieces proportional to m_V^{-1}; the unphysical scalar matrix elements are obviously harmless in the high-energy limit. In this way, ET provides remarkable insight into the nature of the subtle divergence cancellations characteristic of the SM (as well as of the other non-abelian gauge theories involving Higgs-Goldstone scalars). It is clear that the formal ET machinery based on the identities (43) is substantially more efficient in proving the tree-level unitarity than straightforward U-gauge calculations, which

become rather involved even for relatively simple processes.

In fact, ET is not only of fundamental importance for proving some general statements within the electroweak theory, but it is also often used for simplifying practical calculations of Feynman diagrams with external massive vector bosons. The aim of the present lecture notes is to provide some background for a possible further study of ET and related topics. As we already mentioned in the Introduction there are other interesting and important aspects of the subject, both on the technical side and in the area of physical applications. However, a corresponding discussion would go beyond the scope of this introductory treatment. The current literature concerning ET is rather rich and the interested reader may find a (presumably incomplete) list e.g. under ref. [16] – [20], in addition to the basic references used in preparing the present text.

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Appendix

Basics of Standard Model

Glashow-Weinberg-Salam (GWS) standard model (SM) of electroweak interactions provides a unification of the parity-violating (V-A) weak force (responsible e.g. for muon decay and mediated by charged massive vector bosons $W^{\pm}$) and the parity-conserving electromagnetic interaction due to the exchange of massless photon γ. It is a non-abelian gauge theory where particle masses are generated via Higgs mechanism.

The gauge group $SU(2) \times U(1)$ corresponds to four vector fields A_{μ}^{a}, $a = 1, 2, 3$ and B_{μ}. Note that *four* vector bosons are needed since we know that an extra vector boson (apart from $W^{\pm}$ and γ) must be introduced in order to accomplish a technically successful electroweak unification without any exotic fermions (such as a heavy electron etc.)

The Higgs mechanism is realized by means of an $SU(2)$ doublet of complex scalar fields

$$\Phi = \begin{pmatrix} \varphi^{+} \\ \varphi^{0} \end{pmatrix} \tag{A.1}$$

(the lower component of (1) is taken to be electrically neutral). It means that four real scalars are involved; of course, it is so because one needs, in accordance with general properties of the Higgs mechanism, three Goldstone bosons in order to get eventually three massive vector bosons. Note that it would not suffice to take a real triplet of scalars since such an option would leave us with 2 massless neutral vector bosons - only $W^{\pm}$ would get a mass in this way. A minimal scalar multiplet is thus a complex doublet (i.e. real quartet).

The last but not least, there are *fermions* (3 generations of leptons and quarks). In order to describe correctly the parity-violating weak interactions and the parity-conserving electromagnetism, the left-handed and right-handed components of fermion fields must transform differently (doublets for L, singlets for R).

The $U(1)$ transformation properties of all matter fields (i.e. scalars and fermions) are defined by the corresponding weak hypercharge which is given by

$$Q = T_3 + Y \tag{A.2}$$

(T_3 is "weak isospin" and Q is electric charge in units of e). As for fermions, we will first restrict ourselves to leptons (electron type). Denote simply

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad , \quad R = e_R \qquad (A.3)$$

Then according to (2) one obviously has

$$Y_L = -\frac{1}{2} \quad , \quad Y_R = -1 \qquad (A.4)$$

The gauge invariant Lagrangian

The gauge invariant Lagrangian of the GWS standard model may be written as consisting essentially of four pieces, namely

$$\mathcal{L}_{GWS} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} \qquad (A.5)$$

Interactions of vector bosons with leptons

Let us begin with $\mathcal{L}_{fermion}$ (for the moment, the fermions are just ν_e and e). It may be written as

$$\mathcal{L}_{fermion} = i\bar{L}\gamma^\mu(\partial_\mu - igA^a_\mu\frac{\tau^a}{2} + \frac{i}{2}g'B_\mu)L + i\bar{R}\gamma^\mu(\partial_\mu + ig'B_\mu)R \qquad (A.6)$$

where $\tau^a,\ a = 1,2,3$ are Pauli matrices and the $g,\ g'$ are two independent coupling constants. Note that gauge invariance of the expression (6) is due to the covariant derivatives (by which we have replaced the ordinary derivatives in the lepton kinetic terms). In writing (6) we have taken into account the values of weak hypercharges in (4). Working out the last expression and introducing the notation

$$W^\pm_\mu = \frac{A^1_\mu \mp iA^2_\mu}{\sqrt{2}} \qquad (A.7)$$

one first reproduces the standard charged current weak interaction

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}\bar{\nu}_L\gamma^\mu q_L W^+_\mu + \text{h.c.} \qquad (A.8)$$

Further, in the diagonal part of (6) (i.e. in the terms involving τ^3 and the unit matrix) neither A^3_μ nor B_μ can be interpreted as the electromagnetic

potential. However, one may introduce new fields A_μ and Z_μ by means of an orthogonal transformation

$$A^3_\mu = \cos\vartheta_W Z_\mu + \sin\vartheta_W A_\mu \qquad B_\mu = -\sin\vartheta_W Z_\mu + \cos\vartheta_W A_\mu \qquad (A.9)$$

(let us stress that such a transformation must be orthogonal so as not to spoil diagonality of the kinetic energy terms coming from $\mathcal{L}_{gauge}$). Using (9) in eq. (6) one is able to reproduce a correct electromagnetic interaction (i.e. such that does not involve the γ_5 and the neutrino field) by choosing the mixing angle ϑ_W (the Weinberg angle) to satisfy

$$\cos\vartheta_W = \frac{g}{\sqrt{g^2+g'^2}} \quad , \quad \sin\vartheta_W = \frac{g'}{\sqrt{g^2+g'^2}} \qquad (A.10)$$

The electromagnetic coupling constant e then comes out to be

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} = g\sin\vartheta_W \qquad (A.11)$$

One thus gets an important constraint on the relative strength of charged-current weak interaction (g) with respect to the electromagnetic coupling. From $e = g\sin\vartheta_W$ one has

$$e < g \qquad (A.12)$$

(note that $e = g$ is excluded as it is not compatible with (10) for $g \neq 0$). The relation (11) or (12) resp. is sometimes called a "unification condition" in the literature. An important consequence of eq. (11) is a lower bound for the W mass. Indeed, taking into account the well known relation

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} \qquad (A.13)$$

(which expresses compatibility of the Fermi-type theory of weak interactions with a model involving the W boson) then using (11) and the definition of the fine structure constant $\alpha = e^2/4\pi$, one gets the result

$$m_W = \left(\frac{\pi\alpha}{G_F\sqrt{2}}\right)^{\frac{1}{2}} \frac{1}{\sin\vartheta_W} \qquad (A.14)$$

Taking in eq. (14) $G_F \doteq 1.166 \times 10^{-5} GeV^{-2}$ and $\alpha \doteq 1/137$, one gets immediately a lower bound

$$m_W > 37 \text{ GeV} \qquad (A.15)$$

It is remarkable that one is able to derive the result (14) leading to the estimate (15) without even mentioning the Higgs mechanism; thus Glashow in his 1961 paper could in fact predict such a lower bound for m_W (but he failed to do so).

For an interaction of the Z boson with leptons (i.e. for a weak neutral current interaction) one then gets the result (a GWS *prediction*)

$$\mathcal{L}_{NC} = \frac{g}{\cos\vartheta_W}\Big[\frac{1}{2}\bar{\nu}_L\gamma^\mu\nu_L + \Big(-\frac{1}{2}+\sin^2\vartheta_W\Big)\bar{e}_L\gamma^\mu e_L + \sin^2\vartheta_W\bar{e}_R\gamma^\mu e_R\Big]Z_\mu \tag{A.16}$$

Note that the coefficients of the individual terms in the square brackets in (16) satisfy a simple rule

$$\varepsilon_f = T_{3f} - Q_f\sin^2\vartheta_W \tag{A.17}$$

(where f stands for ν_L, e_L or e_R).

Of course, now one should also add the other lepton types μ and τ (which is trivial if a possible lepton mixing is ignored) and, moreover, one has to include 3 generations of quarks. The incorporation of quarks will be described briefly somewhat later. Here let us emphasize that the sector of lepton-vector boson interactions is nowadays the best tested part of SM.

Vector boson self-interactions

Another part of the SM Lagrangian which is generally considered quite trustworthy at present (although its precise experimental tests still lie ahead of us) is the sector of vector boson self-interactions, i.e. the term $\mathcal{L}_{gauge}$ in (5). Let us now summarize some familiar facts concerning the construction of this sector. The $\mathcal{L}_{gauge}$ may be written as

$$\mathcal{L}_{gauge} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{A.18}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc}A^b_\mu A^c_\nu \tag{A.19}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{A.20}$$

Let us remark that gauge invariance of the non-abelian part of (18) is most transparent if one employs the definition of the $F^a_{\mu\nu}$ in terms of a commutator of $SU(2)$ covariant derivatives, namely

$$-igF_{\mu\nu} = [D_\mu,\ D_\nu] \tag{A.21}$$

where

$$F_{\mu\nu} = F^a_{\mu\nu} T^a \qquad (A.22)$$

and

$$D_\mu = \partial_\mu - igA_\mu \qquad (A.23)$$

with $A_\mu = A^a_\mu T^a$; the T^a are $SU(2)$ generators (e.g. $T^a = \frac{1}{2}\tau^a,\ a = 1, 2, 3$). The first term in eq. (18) may be then recast as

$$-\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} = -\frac{1}{2} \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu}) \qquad (A.24)$$

(taking into account $Tr(T^a T^b) = \frac{1}{2}\delta^{ab}$) and this makes the gauge invariance obvious, since the $F_{\mu\nu}$ is transformed covariantly owing to (21).

While the abelian part of (18) is of course just a kinetic term for the $U(1)$ gauge field B_μ, the non-abelian part produces, beside kinetic terms for the A^a_μ, also some specific self-interactions (trilinear and quadrilinear in gauge potentials). When the expression (18) is recast in terms of physical vector boson fields $W^\pm_\mu,\ Z_\mu$ and A_μ (see (7) and (9)) one gets

$$\mathcal{L}_{gauge} = -\frac{1}{2} W^-_{\mu\nu} W^{+\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} + \mathcal{L}_{VVV} + \mathcal{L}_{VVVV} \qquad (A.25)$$

where the notation in the kinetic terms should be self-explanatory and the interactions have the following form (the V is a generic symbol for any vector boson, i.e. $W^\pm,\ Z$ or γ)

$$\mathcal{L}_{VVV} = -ig(W^0_\mu W^-_\nu \overleftrightarrow{\partial^\mu} W^{+\nu} + W^-_\mu W^+_\nu \overleftrightarrow{\partial^\mu} W^{0\nu} + W^+_\mu W^0_\nu \overleftrightarrow{\partial^\mu} W^{-\nu}) \qquad (A.26)$$

$$\mathcal{L}_{VVVV} = -g^2 \Big[\frac{1}{2}(W^-.W^+)^2 - \frac{1}{2}(W^-)^2(W^+)^2 + (W^-.W^+)(W^0)^2 - (W^-.W^0)(W^+.W^0)\Big] \qquad (A.27)$$

where $W^0_\mu = A^3_\mu = \cos\vartheta_W Z_\mu + \sin\vartheta_W A_\mu$, and an obvious shorthand notation for Lorentz scalar products has been used in (27). It is interesting to observe that the expressions (26) and (27) comprise just some particular types of the vector boson interactions, namely the triple vector boson couplings $WW\gamma$ and WWZ and the quartic couplings $WWWW$, $WWZZ$, $WWZ\gamma$ and $WW\gamma\gamma$. For instance, there are no direct triple interactions like ZZZ

etc. (technically this is due to antisymmetry of the $SU(2)$ structure constants $f^{abc} = \varepsilon^{abc}$) and there are no direct quartic couplings like $Z\gamma\gamma\gamma$ etc. (these may be induced in higher orders).

Higgs sector

The least understood part of the SM is its "Higgs sector" or, in other words, the sector responsible for the electroweak symmetry breaking and for generating particle masses. Let us start with the term $\mathcal{L}_{Higgs}$ in (5). This is obtained by replacing ordinary derivatives in scalar kinetic energy by covariant derivatives and it also includes a "potential" of the Goldstone type, i.e.

$$\mathcal{L}_{Higgs} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi) \tag{A.28}$$

where

$$D_\mu = \partial_\mu - igA^a_\mu \frac{\tau^a}{2} - \frac{i}{2} g' B_\mu \tag{A.29}$$

(notice that in (29) we have used $Y_\Phi = \frac{1}{2}$). The "potential" $V(\Phi)$ has a familiar form

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \tag{A.30}$$

The V is minimized for constant Φ_0 such that

$$\Phi_0^\dagger \Phi_0 = \frac{v^2}{2} \tag{A.31}$$

where

$$v = \mu / \sqrt{\lambda} \tag{A.32}$$

The Φ may be written as (a simple exercise in matrix multiplication)

$$\Phi = e^{i\xi^a \tau^a} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H) \end{pmatrix} \tag{A.33}$$

where H is (massive) Higgs boson field ($m_H^2 = 2\lambda v^2$) and the $\xi^a,\ a = 1, 2, 3$ represent the would-be Goldstone bosons. These can be gauged away, setting thus effectively $\xi^a = 0$; this is equivalent to the corresponding $SU(2)$ gauge transformation

$$\Phi \rightarrow \Phi^{(U)} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H) \end{pmatrix} \tag{A.34}$$

fixing the so-called unitary gauge (or simply U-gauge).

From $\mathcal{L}_{Higgs}$ one then gets first of all mass terms for the $W^{\pm}$ and Z bosons (by combining the constant shift v of the Higgs field with the gauge fields from covariant derivatives). The resulting values of the W and Z masses are

$$m_W = \frac{1}{2} g v \,, \qquad m_Z = \frac{1}{2}(g^2 + g'^2)^{1/2} v \tag{A.35}$$

i.e. one has

$$m_W / m_Z = \cos\vartheta_W \tag{A.36}$$

Note that the existence of such a relation is closely related to the fact that Φ is a $SU(2)$ *doublet*. Let us also remark that from $m_W = \frac{1}{2} g v$ and from $G_F/\sqrt{2} = g^2/8m_W^2$ one may get easily

$$v = (G_F\sqrt{2})^{-1/2} \doteq 246 \text{ GeV} \tag{A.37}$$

Further, using (34) in $\mathcal{L}_{Higgs}$ one gets interactions of the type WWH, ZZH, $WWHH$ and $ZZHH$ as well as Higgs boson self-interactions (cubic and quartic). The relevant coupling constants are, for example

$$g_{WWH} = g m_W \tag{A.38}$$

$$g_{WWHH} = \frac{1}{4} g^2 \tag{A.39}$$

$$g_{HHHH} = -\frac{1}{4}\lambda, \qquad \lambda = \frac{G_F m_H^2}{\sqrt{2}} \tag{A.40}$$

The last but not least, there is a Yukawa-type interaction of the Higgs doublet Φ with leptons. It may be written in an obviously $SU(2)$ invariant form as

$$\mathcal{L}_{Yukawa} = -h_e \bar{L}\Phi R + \text{h.c.} \tag{A.41}$$

(it is not difficult to verify that the last expression is also $U(1)$ invariant). The h_e is a coupling constant which may be easily related to other relevant physical parameters. Indeed, using (34) in (41) one gets immediately a mass term for the electron, with

$$m_e = h_e \frac{v}{\sqrt{2}} \tag{A.42}$$

Similarly one obtains a scalar Yukawa interaction of the Higgs boson with electron

$$\mathcal{L}_{eeH} = g_{eeH}\bar{e}eH \tag{A.43}$$

with

$$g_{eeH} = -\frac{g}{2}\frac{m_e}{m_W} \tag{A.44}$$

(in arriving at (44) one has to use (42) and take into account that $v = 2m_W/g$ - see (35)). Eq. (44) thus expresses the well-known dependence of the H couplings to fermions on fermion masses, characteristic for the minimal SM Higgs sector.

Full fermionic sector of the standard model

The present-day SM incorporates 3 generations of leptons and quarks. For simplicity, in what follows we will neglect a possible mixing in the lepton sector as well as neutrino masses. The basic "building blocks" of the lepton sector are then the following:
$SU(2)$ doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad : \quad \text{weak hypercharge } Y_L^{(l)} = -\tfrac{1}{2}$$

$SU(2)$ singlets

$$e_R, \quad \mu_R, \quad \tau_R \quad : \quad \text{weak hypercharge } Y_R^{(l)} = -1$$

The quark sector is built as follows:
$SU(2)$ doublets

$$\begin{pmatrix} u_0 \\ d_0 \end{pmatrix}_L, \quad \begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_L, \quad \begin{pmatrix} t_0 \\ b_0 \end{pmatrix}_L \quad : \quad \text{weak hypercharge } Y_L^{(u)} = \tfrac{1}{6}$$

$SU(2)$ singlets

$$u_{0\,R}, \quad c_{0\,R}, \quad t_{0\,R} \quad : \quad \text{weak hypercharge } Y_R^{(u)} = \tfrac{2}{3}$$
$$d_{0\,R}, \quad s_{0\,R}, \quad b_{0\,R} \quad : \quad \text{weak hypercharge } Y_R^{(d)} = -\tfrac{1}{3}$$

The quark fields labelled by a subscript zero are not, in general, identical with mass eigenstates. Physical quark masses arise from general Yukawa interactions similarly to the lepton case; however, since all quarks are a priori

assumed to be massive, the Yukawa couplings produce a general mass matrix that has to be diagonalized in order to define the mass eigenstates. Such a diagonalization provides in turn a natural description of the mixing in quark sector.

When dealing with the quark sector it is important to notice first that in order to give masses both to u-type quarks (with charge 2/3) and to d-type ones (with charge $-1/3$) via Yukawa interactions one has to employ, beside the Higgs doublet Φ, also a conjugate object $\widetilde{\Phi}$ defined as

$$\widetilde{\Phi} = i\tau_2\Phi^* \qquad (A.45)$$

(prove that (45) is indeed a doublet with respect to $SU(2)$). Introducing a general Yukawa-type interaction for quarks, then through the Φ one gets a general mass matrix (not necessarily hermitean) for d-type quarks

$$\mathcal{L}_{mass} = -(\overline{d_{0\,L}},\ \overline{s_{0\,L}},\ \overline{b_{0\,L}})M \begin{pmatrix} d_{0\,R} \\ s_{0\,R} \\ b_{0\,R} \end{pmatrix} + \text{h.c.} \qquad (A.46)$$

and analogously (through the $\widetilde{\Phi}$) another mass matrix $\widetilde{M}$ for u-type quarks.

Now, every non-singular complex square matrix M or $\widetilde{M}$ may be diagonalized by means of a *biunitary* transformation, i.e.

$$\begin{aligned} M &= \mathcal{U}^\dagger \mathcal{M} \mathcal{V} \qquad (A.47) \\ \widetilde{M} &= \widetilde{\mathcal{U}}^\dagger \widetilde{\mathcal{M}} \widetilde{\mathcal{V}} \end{aligned}$$

where $\mathcal{U}$, $\mathcal{V}$ and $\widetilde{\mathcal{U}}$, $\widetilde{\mathcal{V}}$ are unitary matrices and $\mathcal{M}$, $\widetilde{\mathcal{M}}$ are diagonal positive definite. Thus, the left-handed components of the "mass eigenstates" may be written as

$$\begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} = \widetilde{\mathcal{U}} \begin{pmatrix} u_{0\,L} \\ c_{0\,L} \\ t_{0\,L} \end{pmatrix}, \qquad \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} = \mathcal{U} \begin{pmatrix} d_{0\,L} \\ s_{0\,L} \\ b_{0\,L} \end{pmatrix} \qquad (A.48)$$

(and similarly for the right-handed components).

The charged-current quark weak interactions involve only left-handed quarks, namely

$$\mathcal{L}_{CC}^{(quark)} = \frac{g}{\sqrt{2}}(\overline{u_{0\,L}},\ \overline{c_{0\,L}},\ \overline{t_{0\,L}})\gamma^\mu \begin{pmatrix} d_{0\,L} \\ s_{0\,L} \\ b_{0\,L} \end{pmatrix} W^{+\mu} + \text{h.c.} \qquad (A.49)$$

The last expression may be now recast in terms of the "mass eigenstates"; using the transformation (48) in (49) one gets

$$\mathcal{L}_{CC}^{(quark)} = \frac{g}{\sqrt{2}}(\overline{u_L},\ \overline{c_L},\ \overline{t_L})\gamma^\mu \widetilde{\mathcal{U}}\mathcal{U}^\dagger \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W^{+\mu} + \text{h.c.} \qquad (A.50)$$

The matrix $\widetilde{\mathcal{U}}\mathcal{U}^\dagger$ appearing in (50) may of course be identified with the famous Cabibbo—Kobayashi—Maskawa matrix V_{CKM}. (Note that the result (50) makes it clear that it would not be physically meaningful to consider a mixing of the u-type and d-type quarks separately; thus the CKM mixing is conventionally assumed to occur among d-type quarks.) Recall that the unitary 3×3 matrix in (50) contains in fact just four physically relevant real parameters, which may be interpreted as three Cabibbo-like angles $\vartheta_i, i = 1, 2, 3$ and one phase δ. As it is well known, a possible non-zero value of the phase δ is supposed to be a source of the CP violation within the standard model. As an instructive exercise for the interested reader, we leave it to prove that in the case of n generations a generalized CKM matrix would involve $(n-1)^2$ physically relevant real parameters ($\frac{1}{2}n(n-1)$ angles plus $\frac{1}{2}(n-1)(n-2)$ phases).

In particular, if $n = 2$, one obtains the familiar Glashow—Iliopoulos—Maiani mixing matrix

$$V_{\text{GIM}} = \begin{pmatrix} \cos\vartheta_C & \sin\vartheta_C \\ -\sin\vartheta_C & \cos\vartheta_C \end{pmatrix} \qquad (A.51)$$

where ϑ_C is the Cabibbo angle.

The good news concerning the above-described picture is that quark weak *neutral currents* remain flavour-diagonal (owing to $\mathcal{U}\mathcal{U}^\dagger = 1$ etc.) — i.e. the (generalized) GIM mechanism works (needless to say, the electromagnetic quark current is flavour-diagonal as well). It is good news indeed since otherwise one would run into phenomenological disaster caused by the notorious "flavour changing neutral currents" (FCNC). At present there are numerous experimental data showing that flavour-changing weak decays conserving the hadronic charge are strongly suppressed in comparison with the corresponding charge-changing decays, so that FCNC interactions must be absent or strongly suppressed. Within the standard model, FCNC are absent at the tree level as we have already remarked; they can only be induced at one-loop

(or higher) level and one may thus understand (even quantitatively, to some extent) the corresponding suppression factors for some typical processes.

To close this section let us add that along with the above-mentioned diagonalization of the quark mass matrices, the Yukawa interactions are diagonalized as well and one gets a pattern of Higgs-quark couplings which is completely analogous to the lepton case. Within SM one thus has, for any fermion–Higgs interaction (cf. (44))

$$\mathcal{L}_{ffH} = -\frac{g}{2}\frac{m_f}{m_W}\overline{f}fH \tag{A.52}$$

References

[1] M. Gell-Mann et al., Phys. Rev. **179** (1969) 1518.

[2] S. Weinberg, Phys. Rev. Lett. **27** (1971) 1688.

[3] J. M Cornwall, D. N. Levin, G. Tiktopoulos, Phys. Rev. **D10** (1974) 1145.

[4] J. S. Bell, Nucl. Phys. **B60** (1973) 427.

[5] C. H. Llewellyn Smith, Phys. Lett. **46B** (1973) 233.

[6] S. D. Joglekar, Ann. Phys. (NY) **83** (1974) 427.

[7] J. Hoej, *Introduction to Electroweak Unification: Standard Model from Tree Unitarity* (World Scientific, Singapore 1994).

[8] C. E. Vayonakis, Lett. Nuovo Cim. **17** (1976) 383.

[9] B. W. Lee, C. Quigg, H. Thacker, Phys. Rev. **D16** (1917) 1519.

[10] M. S. Chanowitz, M. K. Gaillard, Nucl. Phys. **B261** (1985) 379.

[11] G. J. Gounaris, R. Kögerler, H. Neufeld, Phys. Rev. **D34** (1986) 3257.

[12] Y.-P. Yao, C.-P. Yuan, Phys. Rev. **D38** (1988) 2237.

[13] J. Bagger, C. Schmidt, Phys. Rev. **D41** (1990) 264.

[14] W. B. Kilgore, Phys. Lett. **B294** (1992) 257.

[15] H.-J. He, Y.-P. Kuang, X. Li, Phys. Rev. Lett. **69** (1992) 2619; Phys. Rev. **D49** (1994) 4842; Phys. Lett. **B329** (1994) 278.

[16] C. Grosse-Knetter, I. Kuss, Z. Phys. **C66** (1995) 95.

[17] C. Grosse-Knetter, Z. Phys. **C67** (1995) 261.

[18] H.-J. He, Y.-P. Kuang, C.-P. Yuan, Phys. Rev. **D51** (1995) 6463; preprint DESY–96–029 (hep-ph/9509278).

[19] D. Espriu, J. Matias, Phys. Rev. **D52** (1995) 6530.

[20] S. D. Willenbrock, Ann. Phys. (NY) **186** (1988) 15.
D. Soper, Z. Kunszt, Nucl. Phys. **B296** (1988) 253.
R. Casalbuoni et al., Phys. Rev. **D39** (1989) 2281.
G. Valencia, S. D. Willenbrock, Phys. Rev. **D42** (1990) 853.
H. Veltman, Phys. Rev. **D41** (1990) 2294.
R. S. Willey, Phys. Rev. **D44** (1991) 3646.
J. F. Donoghue, J. Tandean, Phys. Lett. **B301** (1993) 372.
W. Kilgore, Phys. Lett. **B323** (1994) 161.
P. B. Pal, Phys. Lett. **B321** (1994) 229.
B. Dutta, S. Nandi, Mod. Phys. Lett. **A9** (1994) 1025.
A. Dobado, J. R. Pelaez, Phys. Lett. **B329** (1994) 469.

[21] M. Peskin, SLAC-PUB-4601 (1988); Proc. SLAC Summer Institute 1988.

[22] J. F. Donoghue et al., *Dynamics of Standard Model* (Cambridge University Press, Cambridge 1992).

[23] G. 't Hooft, Nucl. Phys. **B35** (1971) 167.

[24] K. Fujikawa, B. W. Lee, A. I. Sanda, Phys. Rev. **D6** (1972) 2923.

[25] D. Bailin, A. Love, *Introduction to Gauge Field Theory* (Institute of Physics, Bristol 1993).

[26] Ta-Pei Cheng, Ling-Fong Li, *Gauge Theory of Elementary Particle Physics* (Clarendon Press, Oxford 1984).

[27] L. H. Ryder, *Quantum Field Theory* (Cambridge University Press, Cambridge 1985).

[28] C. Becchi, A. Rouet, R. Stora, Comm. Math. Phys. **42** (1975) 127; Ann. Phys. (NY) **98** (1976) 287.

[29] I. V. Tyutin, preprint FIAN No. 39 (1975) (unpublished).

[30] L. Baulieu, Phys. Rep. **129**, No. 1 (1985) 1.

[31] T. Kugo, I. Ojima, Phys. Lett. **73B** (1978) 459.

[32] T. Kugo, I. Ojima, Suppl. Prog. Theor. Phys. **66** (1979) 1.

[33] N. Nakanishi, I. Ojima, *Covariant Operator Formalism of Gauge Theories and Quantum Gravity* (World Scientific, Singapore 1990).

[34] B. W. Lee, J. Zinn-Justin, Phys.Rev. **D5** (1972) 3155.